**Power Series**

**Definition:**

**A power series about**  is a series of the form

  (1)

**A power series about**  is a series of the form

  (2)

in which the center and the coefficients  are constants.

**Remark:**

Recall that the Ratio Test applies to series with nonnegative terms.

**Example:**

For what values of do the following power series converge ?

1. 
2. 
3. 
4. 
5. 
6. 

**Solution:**

(1) 

This is geometric series with first term  and ratio , then the series converges for  and its sum .

(2) 

This is geometric series with first term  and ratio , then the series converges for  and its sum .

(3) 

Apply the ratio test to the series 



The series  is converges for . Then the series  is absolutely convergence for .

The series  diverges for , because the condition

 ( if )

.

If , the alternating series  converges because it satisfies all three conditions of the alternating series test.

If , the series 

diverges because, the series  diverges (*p*-series).

Then from above the series  converges for .

(4) 

Apply the ratio test to the series 



The series  is converges for . Then the series  is absolutely convergence for .

The series  diverges for , because the condition

 ( if )

.

If , the alternating series  converges because it satisfies all three conditions of the alternating series test.

If , the series 

converges because it satisfies all three conditions of the alternating series test.

Then from above the series  converges for .

(5) 

Apply the ratio test to the series 

 for every .

Then the series  is absolutely convergence for all .

(6) 

Apply the ratio test to the series 

 for every except .

Then the series  diverges for all except .

**The Radius of Convergence of a Power Series**

**Theorem:**

The convergence of the series is described by one of the following three cases:

1. There is a positive number such that the series diverges for with but converges absolutely for with *.* The series may or may not converge at either of the endpoints and .
2. The series converges absolutely for every  ().
3. The series converges at and diverges elsewhere ().

is called the radius of convergence of the power series, and the interval of radius 

centered at is called the interval of convergence.

**Remark:**

The interval of convergence may be open, closed, or half-open, depending on the particular series. At points with *,* the series converges absolutely. If the series converges for all values of *,* we say its radius of convergence is infinite. If it converges only at *,* we say its radius of convergence is zero.

**Example:**

Find the series’ radius and interval of convergence of the following power series.

(1)  (2)  (3)  (4) 

(5)  (6)  (7) .

**Solution:**

(1) 

This is geometric series with first term  and ratio , then the series converges for  .

Then the radius and the interval of convergence is .

(2) 

Apply the ratio test to the series 



The series  is converges for . Then the series  is absolutely convergence for .

.

When , the series 

converges .

When , the series 

diverges .

Then the radius is  and the interval of convergence is .

(3) 

This is geometric series with first term  and ratio , then the series converges for  .

Then the radius and the interval of convergence is .

 (4) 

Apply the ratio test to the series 





The series  is converges for . Then the series  is absolutely convergence for .

When , the series  is converges because it satisfies all three conditions of the alternating series test.

When , the series  is converges because  (-series). Then radius is  and the interval of convergence is .

(5) 

Apply the ratio test to the series 



 

The series  is converges for . Then the series  is absolutely convergence for .

When , the series  is converges because it satisfies all three conditions of the alternating series test.

When , the series  is diverges because  (-series). Then radius is  and the interval of convergence is .

(6) 

Apply the ratio test to the series 

 for every .

Then the series  is absolutely convergence for all . Then the radius is  and the series converges for all .

(7) 

Apply the ratio test to the series 



 

  for every except .

Then the series  diverges for all except . Then the radius is  and the series converges only for .